EXCITATION OF A PHASE-TRANSFORMATION WAVE IN A METAL UNDER THE ACTION OF A SQUARE SURFACE HEAT PULSE

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We approximate the phase-transformation wave excited in a metal by the action of a square surface heat pulse when its surface density is given and the time of action is varied.

In the present study we consider the problem of calculating a diagram showing the displacement of the phase boundaries of a metal when it is acted upon by square heat pulses which differ in density of heat flux F and duration of the action t but have equal values of the parameter

$$W = Ft = \text{const}. \tag{1}$$

Figure 1 shows one of these diagrams for copper. The diagram expresses the final position (the final displacement) of the phase boundaries of vaporization and melting at the conclusion of the action of square pulses of various duration t for a given value of the parameter W = 1 J/mm^2.

The phase-transformation wave excited by an individual pulse is described by an essentially nonlinear heat-conduction problem. Using its numerical solution by means of computers, we obtained in the present study a generalization of the numerical data and that enables us to calculate almost completely on the diagram both the vaporization curve y_{vap} and the melting curve y_{me}. The latter is not amenable to calculation only between points 1 and 2. It is quite obvious that there cannot be any extrema between them, and therefore the graphical interpolation of this region, carried out as a monotonic continuation of the well-grounded segments of the y_{me} curve, is entirely acceptable from the viewpoint of accuracy both for purposes of estimation and for more rigorous problems.

As a result of the graphical interpolation of a small segment of the y_{me} curve, we can calculate completely the diagram showing the final position (final displacement) of the phase boundaries for any metal over a wide range of values of the heat-pulse parameters W and t.

In [1] we formulated and solved in a one-dimensional approximation the most general thermophysical problem with two movable phase boundaries — the melting front and the vaporization front. An analysis of this problem shows that under the action of the heat flux the system first develops transition phenomena, and when these are completed, the process enters a stationary regime; the temperature and velocity of the vaporization front are stabilized and take on the values T and v, while the velocity of the melting front approaches and coincides with the velocity of the vaporization front, and a stationary distance x_{me} is established between the two phase boundaries. For a stationary regime of this problem, we obtained the following solution:

$$F = v[r_V + L_V + C_V(T - T_0)];\ x_{\text{me}} = \frac{a}{v} \ln \frac{T + (L_V/C_V) - T_0}{T_{\text{me}} + (L_V/C_V) - T_0}. \tag{2}$$

In the present problem the kinetics of the vaporization are taken into account on the basis of Frenkel's theory and it is assumed that $v = v_0 \exp(-T_m/T)$, where v_0 is a value of the same order as the velocity of sound in metal; $T_m = \varepsilon/k$ (ε is the binding energy of an atom of the metal).

If melting is precluded, then from the simpler one-dimensional model problem, in which the vaporization kinetics are also taken on the basis of Frenkel's theory [2, 3], we obtain

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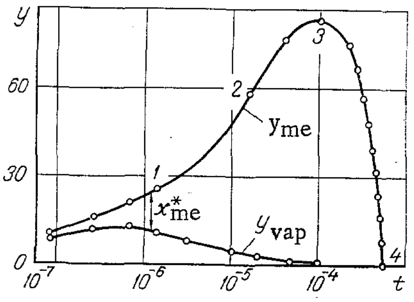

Fig. 1

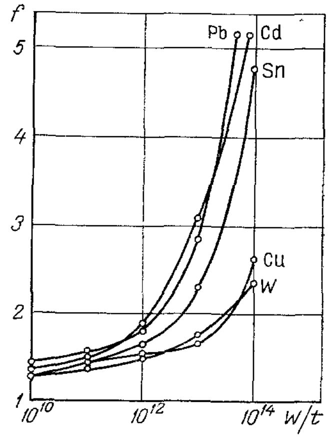

Fig. 2

Fig. 1. Diagram showing the final displacement of the phase boundaries y_{vap} (μm) and y_{me} (μm) for copper as functions of the time of action t (sec) for W = 1 J/mm².

Fig. 2. Variation of f as a function of W/t (J/mm²·sec) for some metals.

an analogous solution for the stationary regime:

$$F = v[r_V + C_V(T - T_0)];\quad x^*_{me} = \frac{a}{v}\ln\frac{T - T_0}{T_{me} + (L_V/C_V) - T_0}. \qquad (3)$$

A comparison of (2) and (3) leads us to some important conclusions. Since for all metals the specific volumetric heat of vaporization r_V is at least one order of magnitude higher than the specific volumetric heat of fusion L_V, it follows that taking account of the melting process has practically no effect on the velocity of the vaporization front and its temperature, so that the latter quantities can be determined in accordance with (3). It is also important that the quantities x_{me} and x^*_{me} also practically coincide, with a difference of only a few percent. This means that the spatial separation of the melting front from the vaporization front, x_{me}, in (2) can be expressed in terms of the quantity x^*_{me} in (3), representing the distance from the vaporization front to the isotherm of the reduced melting temperature ($T^*_{me} = T_{me} + L_V/C_V$).

Thus, (3) can be used with sufficient justification for calculating the diagram of the final displacement both of the vaporization front and of the melting front.

In addition to (3), the basis of the calculation also includes two equations for the same thermophysical problem with the melting front precluded:

$$\tau_{tr} = \frac{a}{v^2};\quad v(\tau) = v\,\mathrm{erf}\left(\frac{2v^2}{a}\tau\right). \qquad (4)$$

These equations relate to the transition process and are the result of a generalization of the numerical data of the machine solution of the problem indicated [4].

Integrating v(τ) in (4) determines the displacement of the vaporization front with respect to its initial position

$$y_{vap} = v\int_0^t \mathrm{erf}\left(\frac{2v^2}{a}\tau\right)d\tau, \qquad (5)$$

where τ is the running time of action of the heat pulse and t is its total duration. For the rest of our discussion, it is essential to transform (5) in such a way as to include the fundamental parameters of the heat pulse, t and W. Defining the density of the heat flux in terms of W and t, in accordance with (1), we can write, on the basis of (3), that

$$\frac{W}{t} = vr_Vf;\quad f = 1 + \frac{C_V}{r_V}\left[\frac{T_m}{\ln(v_0 r_V ft/W)} - T_0\right].$$

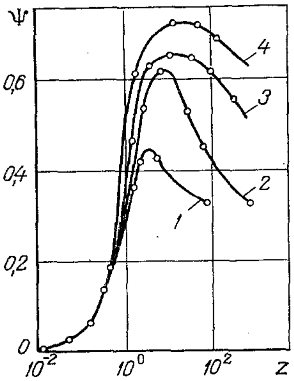

Fig. 3

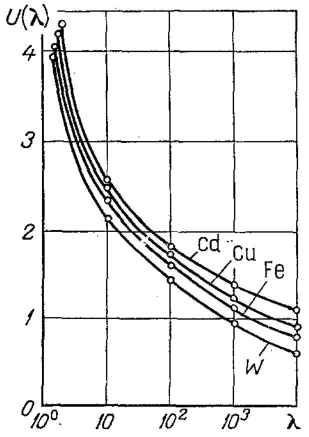

Fig. 4

Fig. 3. Variation of Ψ as a function of z for copper for various values of W: 1) W = 0.1; 2) 1; 3) 10; 4) 100 J/mm².

Fig. 4. Variation of U as a function of λ for some metals.

given parameters W and t. The nature of the variation of f as a function of W/t is shown in Fig. 2 for a number of metals. It can be seen from the figure that f depends weakly on W/t. As W/t varies by several orders of magnitude, the function f changes only by a few units.

Taking account of (6) and carrying out further transformations, we can represent (5) in the form

$$y_{\text{vap}} = \frac{W}{r_v}\Psi, \quad \Psi = \frac{I}{f}, \tag{7}$$

where Ψ is a certain dimensionless function which is equal, in turn, to the ratio of two other dimensionless functions, from which

$$I = I(z) = \frac{1}{z}\int_0^z \operatorname{erf}\varphi\, d\varphi; \quad \varphi = \frac{2W^2\tau}{af^2r_v^2t^2}; \quad z = \frac{2W^2}{af^2r_v^2t}. \tag{8}$$

The sequence of the calculation of y_{vap} is the following: for the given parameters W and t, we determine f from (6); then we determine z and I(t) in accordance with (8); and finally, we determine Ψ from (7). This calculation procedure ensures that we can completely calculate the curve of the final position of the vaporization front y_{vap} on the phase-boundary diagram. There will be no segments of the curve which are not amenable to calculation. This applies both to pulses which bring the process into a stationary regime, when $t > \tau_{\text{tr}}$, and to pulses which do not bring it into this regime when $t < \tau_{\text{tr}}$, where τ_{tr} is the duration of the transitional process in accordance with (4).

In Fig. 3 we show the function Ψ = Ψ(z) for Cu. As can be seen from the figure, this function has clearly defined maxima which depend on the value of the parameter W. For a given value W = const we find that as the parameter t increases, z will decrease and the function Ψ will approach zero. The properties we have found for the function Ψ mean that the curve of the final position of the vaporization front $y_{\text{vap}} = y_{\text{vap}}(t)$ has a maximum and approaches zero asymptotically as t increases, which is shown in Fig. 1. The existence of a maximum on the y_{vap} curve was noted in [4-7].

The position of the melting front with respect to the vaporization front is defined with sufficient accuracy by the quantity x^*_{me} in Eq. (3), which, after changing to the pulse parameters W and t, we can represent in the form

$$x^*_{\text{me}} = \frac{ar_v ft}{W}\cdot U, \tag{9}$$

where the dimensionless function U = U(λ) is defined by the expression

TABLE 1. Values of the Functions η, β_1 and β_2

η	$t_2 \leqslant t \leqslant t_3$	$t_3 \leqslant t \leqslant t_4$	η	$t_2 \leqslant t \leqslant t_3$	$t_3 \leqslant t \leqslant t_4$
	β_1	β_2		β_1	β_2
0	0	5,73	0,6	0,078	3,41
0,1	0,001	5,37	0,7	0,133	2,98
0,2	0,004	5,00	0,8	0,224	2,51
0,3	0,010	4,61	0,9	0,380	1,98
0,4	0,024	4,25	1	1	1
0,5	0,046	3,83			

$$U(\lambda)=\ln\frac{(T_m/\ln\lambda)-T_0}{T^*_{\text{me}}-T_0};\quad \lambda=\frac{v_0 r_V f t}{W}. \tag{10}$$

Figure 4 shows the variation of U as a function of λ for a number of metals. As can be seen from the figure, this function depends weakly on λ. As λ changes by several orders of magnitude, U changes by only a few units.

The sequence of the calculations to obtain x^*_{me} is the following: for given values of W and t, we determine f from (6); then we determine λ and $U(\lambda)$ in accordance with (10); and finally, by (9), we find x^*_{me}.

The final position of the melting front with respect to the initial position of the vaporization front is given by the sum $y_{\text{me}} = y_{\text{vap}} + x^*_{\text{me}}$, which, on the basis of (6)-(10), leads to the expression

$$y_{\text{me}}=\frac{W}{r_V}\,\frac{I(z)}{f}+\frac{a r_V t}{W}\,fU(\lambda). \tag{11}$$

This expression is applicable only to a stationary regime, when $t \leqslant \tau_{\text{tr}}$. The limiting case $t = \tau_{\text{tr}} = t_1$ in Fig. 1 corresponds to the point 1. From (4) and (6) we obtained the following expression for t_1

$$t_1 = W^2/a r_V^2 f^2. \tag{12}$$

This is transcendental, since $f = f(W/t)$, but it is completely solvable. Thus, the curve of the final position of the vaporization front $y_{\text{me}} = y_{\text{me}}(t)$ can be calculated, in accordance with (11), in the entire region $t < t_1$, including the case $t = t_1$.

For a given W and a long pulse duration, intensive vaporization is not stimulated, and the transitional process becomes delayed, so that the condition $\tau_{\text{tr}} \gg t$ is satisfied. If τ_{tr} is one order of magnitude higher than the duration of the pulse ($\tau_{\text{tr}} = 10t$), then the condition $y_{\text{me}} = y_{\text{vap}} + x^*_{\text{me}} \simeq x^*_{\text{me}}$ is satisfied, since $y_{\text{vap}} \ll x^*_{\text{me}}$.

In this case the vaporization may be disregarded, and the thermophysical process may be approximated as the excitation of a melting wave. This means that in the diagram of the final displacement of the phase boundaries (Fig. 1) the segment of the y_{me} curve indicated by the characteristic points 2, 3, 4 can be determined with sufficient accuracy from the position of the isotherm of the reduced melting point, in full accordance with the calculation method we described in [8]. To make use of this method, we must distinguish three values of the duration of the square pulse — t_2, t_3, and t_4 — corresponding to the indicated characteristic points 2, 3, 4:

$$t_2 = 10\frac{W^2}{a r_V^2 f^2};\quad t_3 = MW^2;\quad t_4 = 5.73MW^2. \tag{13}$$

The equation for t_2 is obtained from (4) and the condition $\tau_{\text{tr}} = 10t_2$ and is solvable even if it is transcendental, since $f = f(W/t)$. The equations for t_3 and t_4 were obtained in [8].

In accordance with [8], the calculation of the segment of the $y_{\text{me}}\|$ curve indicated by the points 2, 3, 4 can be carried out on the basis of the equations

$$y_{\text{me}} = \eta NW;\quad t = \beta_1 MW^2 \quad (t_2 \leqslant t \leqslant t_3); \tag{14}$$

$$y_{\text{me}} = \eta NW;\quad t = \beta_2 MW^2 \quad (t_3 \leqslant t \leqslant t_4), \tag{15}$$

where N and M are quantities related to the thermophysical characteristics of the metals.

The functions η, β_1, and β_2 appearing in (14) and (15) are dimensionless, and their values are shown in Table 1.

The data in the first and last columns of these tables, for the functions η and β_2, enables us to calculate on the basis of (15) the segment of the y_{me} curve from the point 4, for which $\eta = 0$ and $\beta_2 = 0.573$ ($y_{me} = 0$, $t_4 = 0.573MW^2$), to the point 3 (the maximum of the y_{me} curve), for which $\eta = 1$ and $\beta_2 = 1$ ($y_{me} = NW$; $t_3 = MW^2$).

If we begin the calculations on the basis of (14) from the bottom numbers of the η and β_1 columns in Table 1, this will give us the calculation of the y_{me} curve in the direction from the point 3 to the point 2. It is also possible to have a case in which the table does not show the data which accurately determine the point 2 (t_2, y_{me2}). The coefficients for the quantities t_2 and y_{me2} may turn out to be between two adjacent rows of the table, the lower of which determines t_2' and y_{me2}', while the upper determines t_2'' and y_{me2}'', where $t_2'' < t_2 < t_2'$, $y_{me2}'' < y_{me2} < y_{me2}'$. Then, resorting to linear interpolation and taking account of the semilogarithmic scale of the diagram, we find for y_{me2}

$$y_{me2} = NW\left[\eta'' + (\eta' - \eta'')\frac{\lg(ar_V^2 f^2 M\beta_1'') - 1}{\lg(\beta_1'/\beta_1'')}\right]. \tag{16}$$

The method described in this article for determining the functions $y_{me} = y_{me}(t)$ and $y_{vap} = y_{vap}(t)$ ensures the complete calculation of the diagram of the final displacement of the phase boundaries of the metal. The diagram illustrates visually and convincingly the fact that square heat pulses coinciding in the value of the parameter W but differing in the duration t of their action can bring about completely different results. When the pulse lasts a long time, it excites an essentially in-depth melting process, whereas when it lasts a short time, we have sublimative destruction with the formation of a small quantity of the liquid phase.

The investigations show that for the same value of the parameter W the diagrams for different metals differ considerably among themselves. A comparison of these diagrams makes clear the specific individuality of the metals with respect to the identical heat-pulse action.

The diagram contains a great deal of generalized information concerning the results produced on a metal by the application of pulsed heat. It is important that this information can be obtained without expensive machine solutions of essentially nonlinear problems approximating the thermophysical processes excited by the heat pulses.

Furthermore, the diagrams provide some guideline information, important for the solution of technological problems connected with the action of heat pulses on metals.

NOTATION

C_V, specific volumetric heat capacity; r_V, specific volumetric heat of vaporization; L_V, specific volumetric heat of fusion; a, thermal conductivity; T_{me}, melting point of the metal; T_m, a temperature which is characteristic of the metal and is determined from the condition $kT_m = \varepsilon$; ε, binding energy of the atom; k, Boltzmann constant; W, surface density of heat-pulse energy; T_0, initial temperature.

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FLOW OF A THIN FILM OF A VISCOUS LIQUID IN A GAS JET

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An approximate formulation is considered for the steady-state wave flow of a thin film of viscous liquid subject to tangential frictional stresses at the boundary. Measurements have been made on the stability limit for droplet detachment, and it has been found that the detachment rate and fractional composition of the droplets are dependent on the boundary conditions.

Vapor—liquid and gas—liquid media are widely used in power systems and various engineering devices, which has led to interest in flows of thin films of liquid at $\mathrm{Re}_{\mathrm{fi}}$ = 5-400, which are characteristic of the natural conditions at the walls of equipment and the corresponding flow region with capillary waves on the surface [1].

There are papers [1, 2] on the basic laws of the wave motion of a thin film for flow in a constant field of mass forces, and these have been extended [3, 4]. However, the scheme used there did not have tangential frictional stresses at the free boundary of the film, so it was impossible to extend the conclusions and equations to the flow of a film in a gas jet, where the viscous interaction between the phases is responsible for tangential frictional stresses τ_0 at the interface.

We consider the case where the film is acted on by a mass force strength j, while the surface is in a gas flow, with the gas pressure constant along the flow direction ($\partial P''/\partial x = 0$); we assume that there is no heat or mass transfer in the wall—film—gas system, while τ_0 is taken as uniformly distributed over the film.

Following [1], the current thickness of the film is specified as

$$\delta = \delta_0 (1 + \varphi), \tag{1}$$

where φ is some function of the coordinate x and time t, which defines the deviation of the film thickness from the mean value δ_0.

We restrict consideration to steady-state wave flow, where any function of φ satisfies

$$\frac{\partial F(\varphi)}{\partial t} = -k \frac{\partial F(\varphi)}{\partial x}, \tag{2}$$

where k is the phase velocity of the waves.

We also assume that δ/λ and a/λ should be small.

With these assumptions, the wave flow can be described by a system of equations consisting of the equation of motion in the Navier—Stokes form:

$$\frac{\partial v'_x}{\partial t} + v'_x \frac{\partial v'_x}{\partial x} + v'_y \frac{\partial v'_x}{\partial y} = -\frac{1}{\rho'} \frac{\partial P'}{\partial x} + j_x + \frac{\mu'}{\rho'} \nabla^2 v'_x \tag{3}$$

and the equation of continuity

$$\frac{\partial (\rho' \bar{v}')}{\partial x} = -\frac{\partial \delta}{\partial t}, \tag{4}$$

where

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